Surds and rationalising the denominator

> **A LEVEL LINKS**
> **Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\dfrac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\dfrac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1   Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2   Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$	
$= 3\sqrt{3}$	**4** Collect like terms

Example 3   Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $=\sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $=7-2$ $=5$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2=49$ **2** Collect like terms: $\quad -\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $\qquad =-\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2}=0$

Example 4   Rationalise $\dfrac{1}{\sqrt{3}}$

$\dfrac{1}{\sqrt{3}}=\dfrac{1}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$ $\quad=\dfrac{1\times\sqrt{3}}{\sqrt{9}}$ $\quad=\dfrac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9}=3$

Example 5   Rationalise and simplify $\dfrac{\sqrt{2}}{\sqrt{12}}$

$\dfrac{\sqrt{2}}{\sqrt{12}}=\dfrac{\sqrt{2}}{\sqrt{12}}\times\dfrac{\sqrt{12}}{\sqrt{12}}$ $\quad=\dfrac{\sqrt{2}\times\sqrt{4\times3}}{12}$ $\quad=\dfrac{2\sqrt{2}\sqrt{3}}{12}$ $\quad=\dfrac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **4** Use $\sqrt{4}=2$ **5** Simplify the fraction: $\quad \dfrac{2}{12}$ simplifies to $\dfrac{1}{6}$

Example 6 Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \dfrac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \dfrac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
$= 3\sqrt{5}-6$	

Practice

1 Simplify.

   a $\sqrt{45}$ **b** $\sqrt{125}$

   c $\sqrt{48}$ **d** $\sqrt{175}$

   e $\sqrt{300}$ **f** $\sqrt{28}$

   g $\sqrt{72}$ **h** $\sqrt{162}$

> **Hint**
>
> One of the two numbers you choose at the start must be a square number.

2 Simplify.

   a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

   c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

   e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

> **Watch out!**
>
> Check you have chosen the highest square number at the start.

3 Expand and simplify.

   a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

   c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\dfrac{1}{\sqrt{5}}$ b $\dfrac{1}{\sqrt{11}}$

c $\dfrac{2}{\sqrt{7}}$ d $\dfrac{2}{\sqrt{8}}$

e $\dfrac{2}{\sqrt{2}}$ f $\dfrac{5}{\sqrt{5}}$

g $\dfrac{\sqrt{8}}{\sqrt{24}}$ h $\dfrac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\dfrac{1}{3-\sqrt{5}}$ b $\dfrac{2}{4+\sqrt{3}}$ c $\dfrac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ b $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

   c $4\sqrt{3}$ **d** $5\sqrt{7}$

   e $10\sqrt{3}$ **f** $2\sqrt{7}$

   g $6\sqrt{2}$ **h** $9\sqrt{2}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$

   c $3\sqrt{2}$ **d** $\sqrt{3}$

   e $6\sqrt{7}$ **f** $5\sqrt{3}$

3 **a** -1 **b** $9-\sqrt{3}$

   c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

4 **a** $\dfrac{\sqrt{5}}{5}$ **b** $\dfrac{\sqrt{11}}{11}$

   c $\dfrac{2\sqrt{7}}{7}$ **d** $\dfrac{\sqrt{2}}{2}$

   e $\sqrt{2}$ **f** $\sqrt{5}$

   g $\dfrac{\sqrt{3}}{3}$ **h** $\dfrac{1}{3}$

5 **a** $\dfrac{3+\sqrt{5}}{4}$ **b** $\dfrac{2(4-\sqrt{3})}{13}$ **c** $\dfrac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 **a** $3+2\sqrt{2}$ **b** $\dfrac{\sqrt{x}+\sqrt{y}}{x-y}$