

Solving simultaneous equations graphically

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

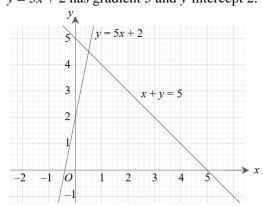
• You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

Example 1 Solve the simultaneous equations y = 5x + 2 and x + y = 5 graphically.

y = 5 - x

y = 5 - x has gradient -1 and y-intercept 5. y = 5x + 2 has gradient 5 and y-intercept 2.



Lines intersect at x = 0.5, y = 4.5

Check:

First equation y = 5x + 2:

$$4.5 = 5 \times 0.5 + 2$$

Second equation x + y = 5:

$$0.5 + 4.5 = 5$$
 YES

YES

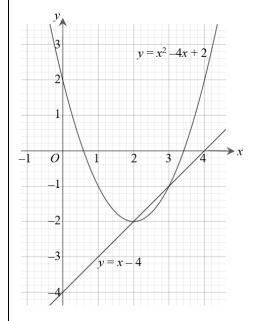
- 1 Rearrange the equation x + y = 5 to make y the subject.
- 2 Plot both graphs on the same grid using the gradients and *y*-intercepts.

- The solutions of the simultaneous equations are the point of intersection.
- 4 Check your solutions by substituting the values into both equations.



Example 2 Solve the simultaneous equations y = x - 4 and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	-1	-2	-1	2



The line and curve intersect at x = 3, y = -1 and x = 2, y = -2

Check:

First equation y = x - 4:

$$-1 = 3 - 4$$

 $-2 = 2 - 4$

YES YES

Second equation $y = x^2 - 4x + 2$:

$$-1 = 3^{2} - 4 \times 3 + 2$$
$$-2 = 2^{2} - 4 \times 2 + 2$$

YES YES

- 1 Construct a table of values and calculate the points for the quadratic equation.
- 2 Plot the graph.
- Plot the linear graph on the same grid using the gradient and y-intercept.
 y = x 4 has gradient 1 and y-intercept -4.

- 4 The solutions of the simultaneous equations are the points of intersection.
- 5 Check your solutions by substituting the values into both equations.

Practice

1 Solve these pairs of simultaneous equations graphically.

a
$$y = 3x - 1$$
 and $y = x + 3$

b
$$y = x - 5$$
 and $y = 7 - 5x$

$$y = 3x + 4$$
 and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a
$$x + y = 0$$
 and $y = 2x + 6$

b
$$4x + 2y = 3$$
 and $y = 3x - 1$

$$\mathbf{c}$$
 $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make *y* the subject.





- 3 Solve these pairs of simultaneous equations graphically.
 - **a** y = x 1 and $y = x^2 4x + 3$
 - **b** y = 1 3x and $y = x^2 3x 3$
 - c y = 3 x and $y = x^2 + 2x + 5$
- **4** Solve the simultaneous equations x + y = 1 and $x^2 + y^2 = 25$ graphically.

Extend

- 5 a Solve the simultaneous equations 2x + y = 3 and $x^2 + y = 4$
 - i graphically
 - ii algebraically to 2 decimal places.
 - **b** Which method gives the more accurate solutions? Explain your answer.





Answers

1 **a**
$$x = 2, y = 5$$

b
$$x = 2, y = -3$$

c
$$x = -0.5, y = 2.5$$

2 **a**
$$x = -2, y = 2$$

b
$$x = 0.5, y = 0.5$$

c
$$x = -1, y = -2$$

3 **a**
$$x = 1, y = 0 \text{ and } x = 4, y = 3$$

b
$$x = -2$$
, $y = 7$ and $x = 2$, $y = -5$

$$\mathbf{c}$$
 $x = -2$, $y = 5$ and $x = -1$, $y = 4$

4
$$x = -3$$
, $y = 4$ and $x = 4$, $y = -3$

5 **a i**
$$x = 2.5, y = -2 \text{ and } x = -0.5, y = 4$$

ii
$$x = 2.41, y = -1.83$$
 and $x = -0.41, y = 3.83$

b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.