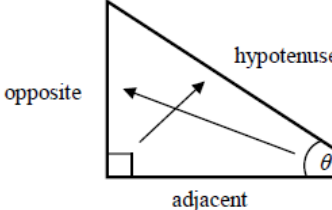# Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
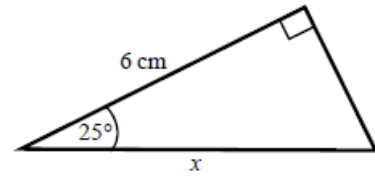
Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.

- In a right-angled triangle:

  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

- The sine, cosine and tangent of some angles may be written exactly.

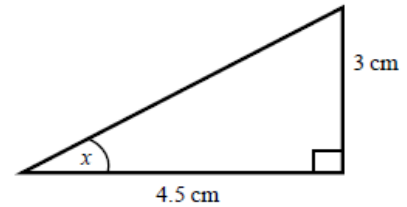|	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1   Calculate the length of side x.
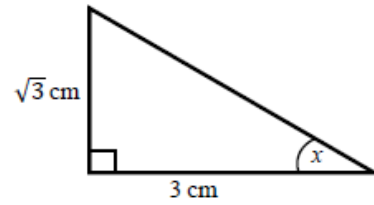Give your answer correct to 3 significant figures.

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\cos 25° = \dfrac{6}{x}$ $x = \dfrac{6}{\cos 25°}$ $x = 6.620\ 267\ 5...$ $x = 6.62$ cm	**1** Always start by labelling the sides. **2** You are given the adjacent and the hypotenuse so use the cosine ratio. **3** Substitute the sides and angle into the cosine ratio. **4** Rearrange to make x the subject. **5** Use your calculator to work out $6 \div \cos 25°$. **6** Round your answer to 3 significant figures and write the units in your answer.

Example 2   Calculate the size of angle x.
Give your answer correct to 3 significant figures.

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\tan x = \dfrac{3}{4.5}$ $x = \tan^{-1}\left(\dfrac{3}{4.5}\right)$ $x = 33.690\ 067\ 5...$ $x = 33.7°$	**1** Always start by labelling the sides. **2** You are given the opposite and the adjacent so use the tangent ratio. **3** Substitute the sides and angle into the tangent ratio. **4** Use $\tan^{-1}$ to find the angle. **5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$. **6** Round your answer to 3 significant figures and write the units in your answer.

Example 3   Calculate the exact size of angle x.
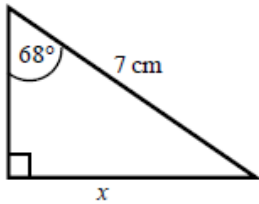

  $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\tan x = \dfrac{\sqrt{3}}{3}$ $x = 30°$	**1** Always start by labelling the sides. **2** You are given the opposite and the adjacent so use the tangent ratio. **3** Substitute the sides and angle into the tangent ratio. **4** Use the table from the key points to find the angle.

Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a
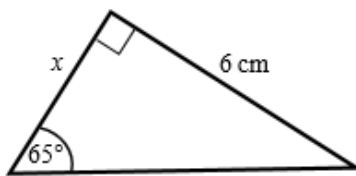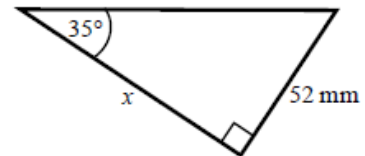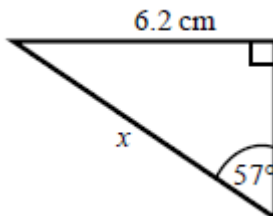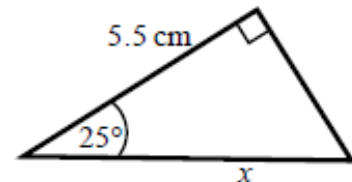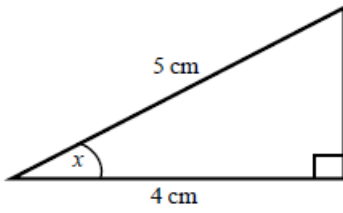

b
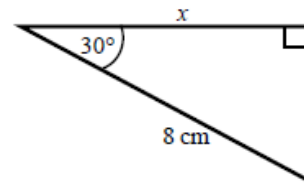
c

d

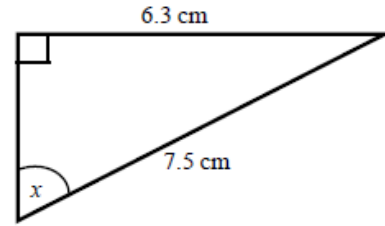**e**

f

2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.
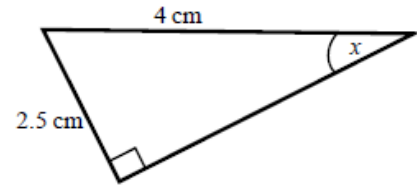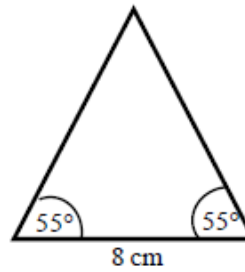
a

b

c

d

3 Work out the height of the isosceles triangle.
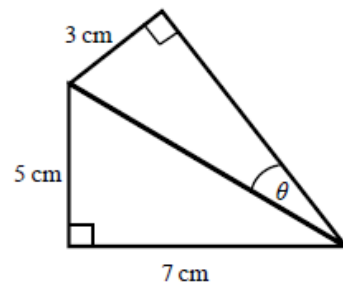Give your answer correct to 3 significant figures.

Hint:

Split the triangle into two right-angled triangles.

4 Calculate the size of angle θ.
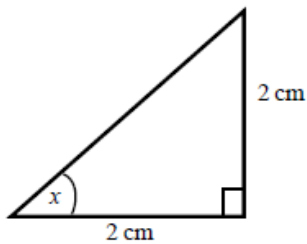Give your answer correct to 1 decimal place.

Hint:

First work out the length of the common side to both triangles, leaving your answer in surd form.
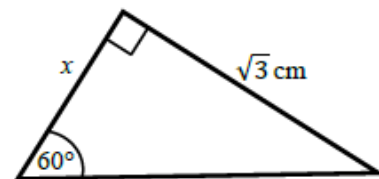

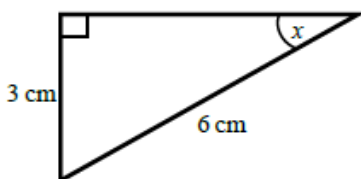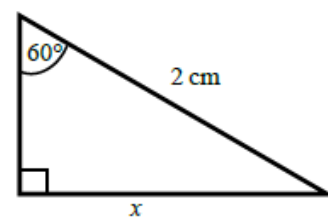**5** Find the exact value of x in each triangle.

a

b

c

d

The cosine rule

Key points

- a is the side opposite angle A.
  b is the side opposite angle B.
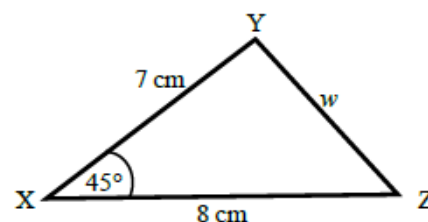 c is the side opposite angle C.
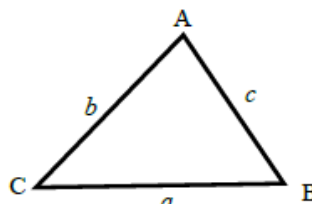
- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.

- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.

- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.
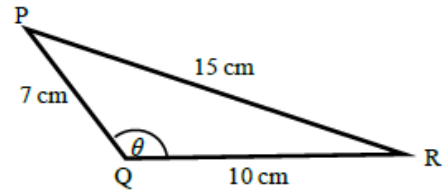
Examples

Example 4   Work out the length of side w.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values a, b and A into the formula.
$w^2 = 33.804\,040\,51...$ $w = \sqrt{33.80404051}$	**4** Use a calculator to find w^2 and then w.
$w = 5.81$ cm	**5** Round your final answer to 3 significant figures and write the units in your answer.

1 Always start by labelling the angles and sides.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

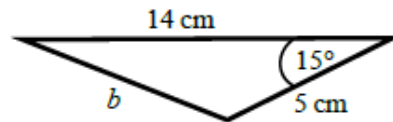|	
	1 Always start by labelling the angles and sides.
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values a, b and c into the formula.
$\cos\theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\,349\ldots$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a

b

c
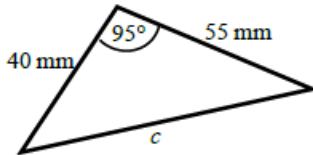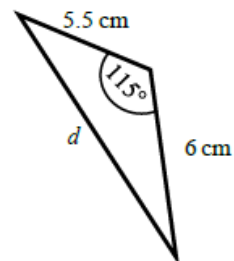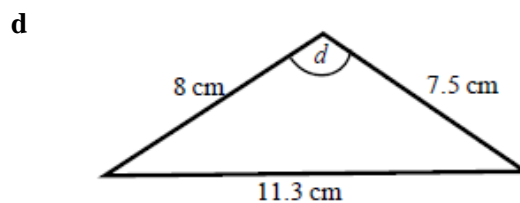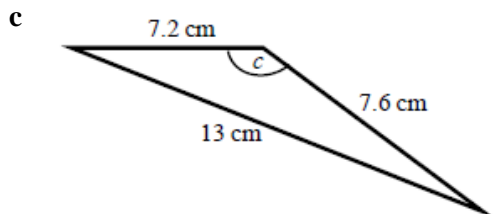

d

7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a

9 cm

a

5 cm

12 cm

b

33 mm

38 mm

36 mm

b

c

7.2 cm

c

7.6 cm

13 cm

d

d

8 cm

7.5 cm

11.3 cm
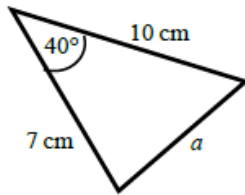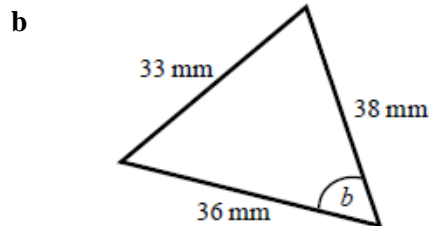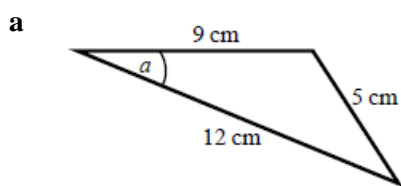
8 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

W

14 cm

X

5 cm

8.5 cm

95°

10 cm

Y

Z
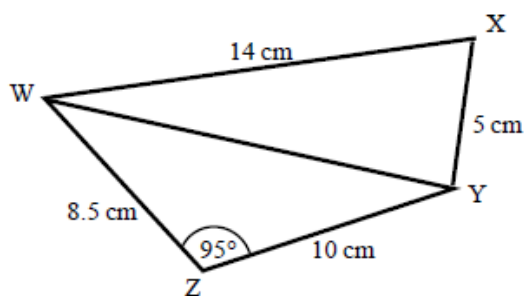
The sine rule

> **A LEVEL LINKS**
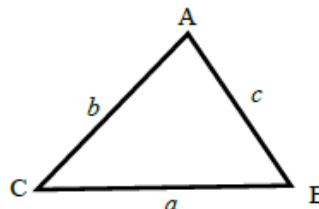> **Scheme of work:** 4a. Trigonometric ratios and graphs
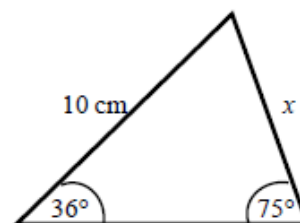> **Textbook:** Pure Year 1, 9.2 The sine rule

Key points

- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.

- To calculate an unknown side use the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.

- To calculate an unknown angle use the formula $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.
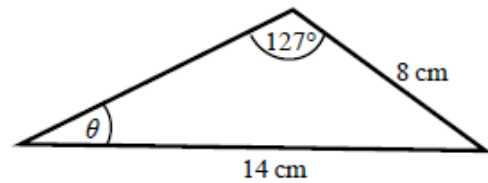
Examples

Example 6 Work out the length of side x.
Give your answer correct to 3 significant figures.

|	
	1 Always start by labelling the angles and sides.
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\dfrac{x}{\sin 36°} = \dfrac{10}{\sin 75°}$	**3** Substitute the values a, b, A and B into the formula.
$x = \dfrac{10 \times \sin 36°}{\sin 75°}$	**4** Rearrange to make x the subject.
$x = 6.09$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\dfrac{\sin\theta}{8} = \dfrac{\sin 127°}{14}$	**3** Substitute the values a, b, A and B into the formula.
$\sin\theta = \dfrac{8\times\sin 127°}{14}$	**4** Rearrange to make $\sin\theta$ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

And to the left of the working:
1 Always start by labelling the angles and sides.

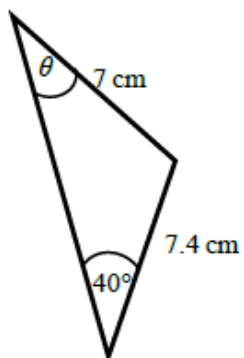## Practice

9   Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.
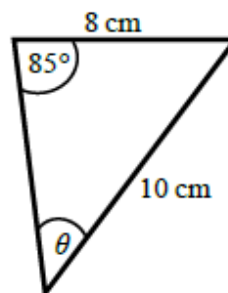
a

b

c

d
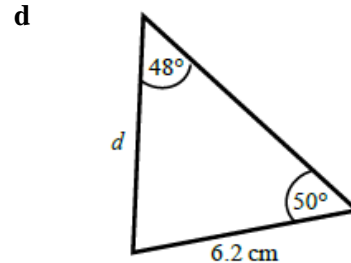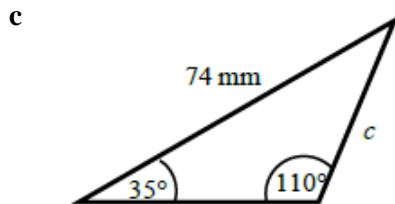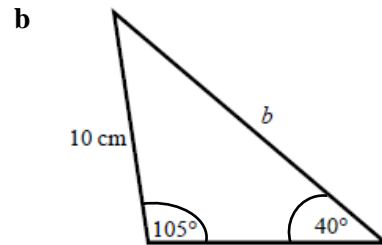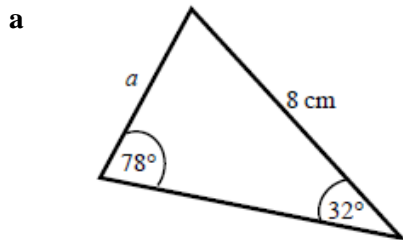
10 Calculate the angles labelled θ in each triangle.
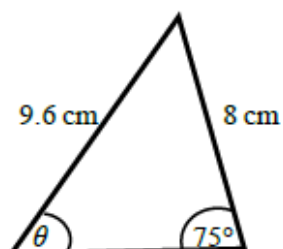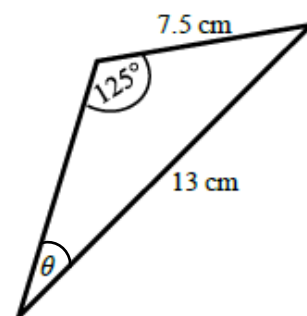Give your answer correct to 1 decimal place.
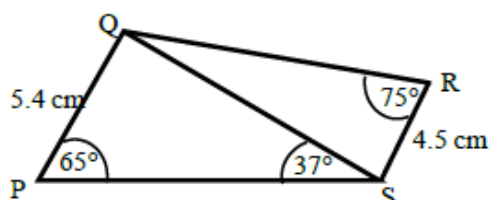
a

b

c

d

11 a Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
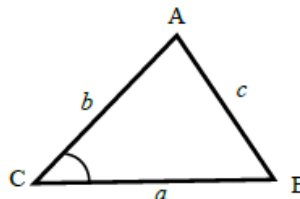Give your answer correct to 1 decimal place.
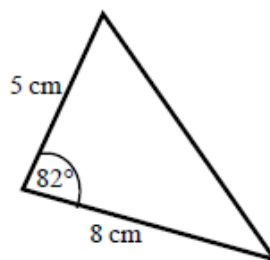
Areas of triangles

Key points

- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

- The area of the triangle is $\frac{1}{2}ab\sin C$.

## Examples

Example 8   Find the area of the triangle.

Area $= \frac{1}{2}ab\sin C$	**1** Always start by labelling the sides and angles of the triangle.
Area $= \frac{1}{2}\times 8\times 5\times \sin 82°$	**2** State the formula for the area of a triangle.
	3 Substitute the values of a, b and C into the formula for the area of a triangle.
Area $= 19.805\,361...$	**4** Use a calculator to find the area.
Area $= 19.8$ cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

Practice

12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

a

7.5 cm
8 cm
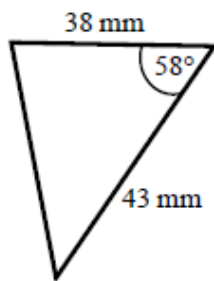37°

b

5.5 cm
115°
7.5 cm

c

38 mm
58°
43 mm
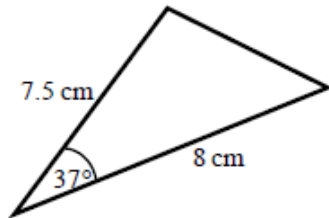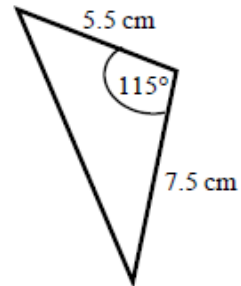
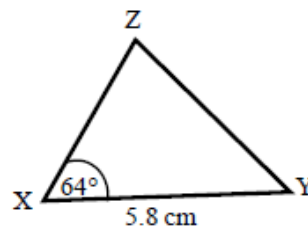**13** The area of triangle XYZ is 13.3 cm².
Work out the length of XZ.

Hint:

Rearrange the formula to make a side the subject.

Z
X
64°
5.8 cm
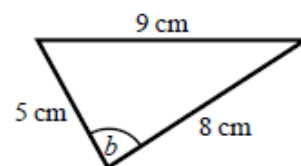Y

Extend

14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.
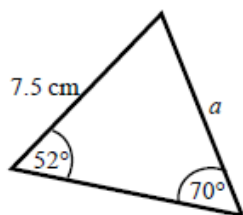
Hint:

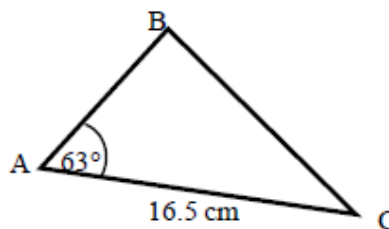For each one, decide whether to use the cosine or sine rule.

a

7.5 cm
a
52°
70°

b

9 cm
5 cm
8 cm
b

c
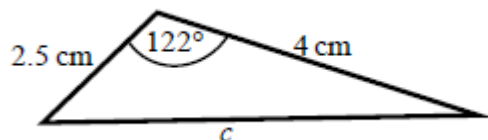
d
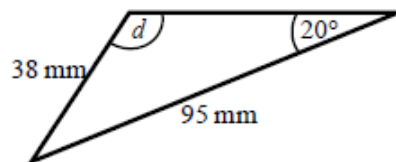
15 The area of triangle ABC is 86.7 cm².
Work out the length of BC.
Give your answer correct to 3 significant figures.

Answers

1 **a** 6.49 cm   **b** 6.93 cm   **c** 2.80 cm
    d 74.3 mm   **e** 7.39 cm   **f** 6.07 cm

2 **a** 36.9°   **b** 57.1°   **c** 47.0°   **d** 38.7°

3 5.71 cm

4 20.4°

5 **a** 45°   **b** 1 cm   **c** 30°   **d** $\sqrt{3}$ cm

6 **a** 6.46 cm   **b** 9.26 cm   **c** 70.8 mm   **d** 9.70 cm

7 **a** 22.2°   **b** 52.9°   **c** 122.9°   **d** 93.6°

8 **a** 13.7 cm   **b** 76.0°

9 **a** 4.33 cm   **b** 15.0 cm   **c** 45.2 mm   **d** 6.39 cm

10 **a** 42.8°   **b** 52.8°   **c** 53.6°   **d** 28.2°

11 **a** 8.13 cm   **b** 32.3°

12 **a** 18.1 cm^2 **b** 18.7 cm^2 **c** 693 mm^2

13 5.10 cm

14 **a** 6.29 cm   **b** 84.3°   **c** 5.73 cm   **d** 58.8°

15 15.3 cm